EDEXCEL NATIONAL CERTIFICATE/DIPLOMA
SCIENCE FOR TECHNICIANS

OUTCOME 3 - ENERGY

TUTORIAL 1 – MECHANICAL WORK, ENERGY AND POWER: WORK

3 Energy

Mechanical work, energy and power: work - energy relationship, gravitational potential energy, kinetic energy, conservation of energy, work done and energy transfer, work done against friction

Heat energy: elementary molecular definition of solids, liquids and gases; gas laws, transfer of heat energy, temperature, temperature scales, change of state, expansion of solids and liquids

You should judge your progress by completing the self assessment exercises. These may be sent for marking at a cost (see home page).

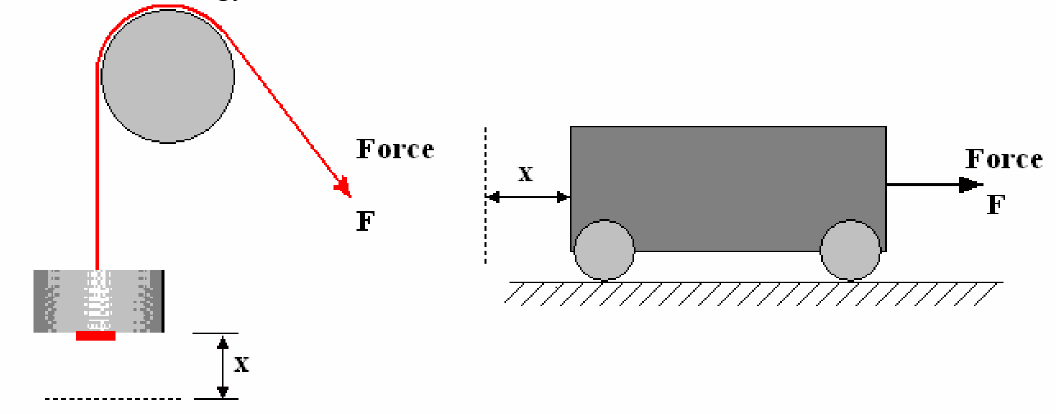
It is not clear whether the syllabus intends the student to cover angular motion also. It would seem to be a lot of material to cover so it is left to the individual to decide whether or not to study problems involving rotation. In order to do them you need to have studied angular motion and moment of inertia.

1. WORK

When a force 'F' moves a distance 'x', the work done 'W' is **W = F x**

In order to do work, you must use an equal amount of energy and since energy cannot be destroyed, it must have been transferred somewhere else (Law of conservation of energy). For example if a mass is raised on a pulley, work is done and the energy of the mass increases as it is lifted. The energy used cannot have been destroyed so it must be stored in the mass as an increase in its potential (gravitational) energy.

Another example is a truck being accelerated along a floor. A force is needed to accelerate the truck and as it moves more and more work is done. The energy used to accelerate the mass becomes stored in it as kinetic energy.



Both the examples show that energy may be transferred to a mass by doing work. It follows that ***ENERGY IS STORED WORK***

2. ENERGY FORMS

POTENTIAL ENERGY

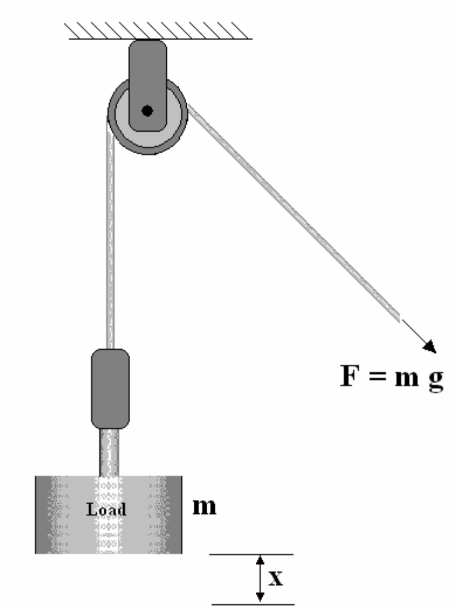
This is the energy stored in a body by virtue of its altitude z and is also called gravitational energy. Consider a mass M kg raised a height z metres against the force of gravity. The weight is Mg.
The work done = weight x distance moved

$$W = Mgz$$

Since this energy cannot be destroyed it is stored in the mass and may be recovered.
The Potential energy is **P.E. = mgz**

Note that 'z' is the S. I. symbol for altitude but 'h' for height and 'x, for distance is also commonly used.

For example if a mass is raised on a simple pulley, work is done and the energy of the mass increases as it is lifted. From the law of conservation of energy, the energy used up cannot have been destroyed so it must be stored in the mass as an increase in its potential (gravitational) energy.



WORKED EXAMPLE No. 1

If the mass being lifted is 200 kg and it is raised 0.6 m, determine the work done and the change in P.E. of the mass.

SOLUTION

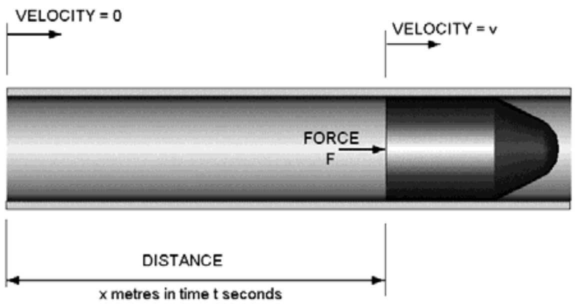
The weight is mg so the force to be overcome is F = 200 x 9.81 = 1962 N

W = 1962 x 0.6 = 1177.2 J

The change in P. E. is the same assuming no energy was wasted.

KINETIC ENERGY

This is the energy stored in a body by virtue of its velocity. Consider a mass, which is at rest. A force is applied to it and it accelerates at a m/s^2 and after t seconds it achieves a velocity of v m/s and has travelled x metres.



From the laws of motion we know that

$F = M a$

$a = v/t \qquad x = vt/2$

The work done is $W = F\,x = M(v/t)(vt/2) = Mv^2/2$

Since the energy used up cannot be destroyed, it is stored in the mass and may be recovered.
The Kinetic Energy is **K.E. = $Mv^2/2$**

WORKED EXAMPLE No. 2

A force of 80 N is used to pull a truck 200 m along a horizontal floor. Determine the work done and the increase in K.E.

SOLUTION

W = F x = 80 x 200 = 16000 J If there was no energy lost due to friction then this must end up as kinetic energy in the truck.

ANGULAR KINETIC ENERGY

A rotating wheel has kinetic energy since all parts of the wheel are moving and have a velocity in a circular path. The corresponding formula is:-

$$K.E. = I\,\omega^2/2$$

ω is the angular velocity of the wheel in radian/sec and I is the moment of inertia. For those not familiar with this concept here is a brief explanation.

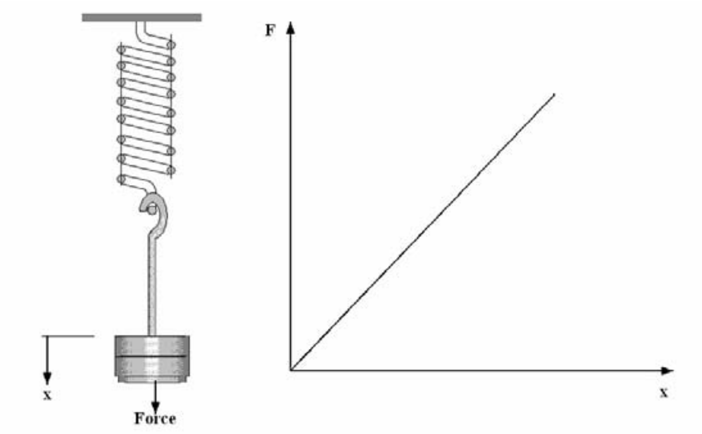
The mass of any wheel is spread around at various radii and so an affective radius is used called the RADIUS OF GYRATION symbol k. If this is known we can calculate I with :-

$$I = m\,k^2$$

For a solid plain disc, k = 0.707R where R is the outer radius.
You also need to know that linear and angular velocities are linked by $v = \omega\, r$ where r is the radius at which v is measured.

STRAIN ENERGY

When an elastic body is deformed and released, it springs back to its original shape and size. In order to deform it, that is, bend it, twist it, squash it or stretch it, work is done. This work is stored in the body as strain energy. Consider a simple spring, which requires F Newton to stretch it 1 m.



The stiffness is K N/m. When the spring is stretched in stages the plot of F against x is a straight line as shown. The gradient is K = F/x and this is Hooke's Law.
The work done is the mean force x distance moved.
The mean force = F/2 W = Fx/2
This work is stored in the spring as strain energy, which also has a symbol of U.

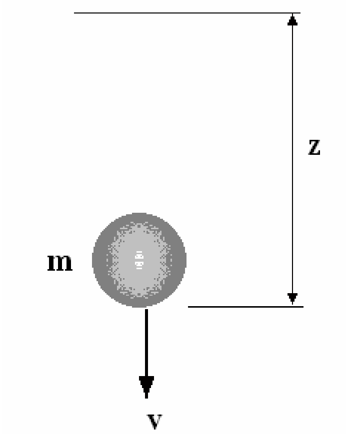
U = Fx/2 for a simple spring.

Note that F/2 is the average force needed to stretch the spring.
Since F = K x then **$U = Kx^2/2$ for a simple spring.**

3. ENERGY CONVERSION

Many engineering problems involve the conversion of energy from one form to another. The law of energy conservation tells us that if no energy is lost to friction then the energy before a change is the same as the energy after a change. This may be used to solve problems. In later problems we shall deal with conversions with energy loss but first let's study simple cases with no losses in the conversion process.

FALLING BODIES

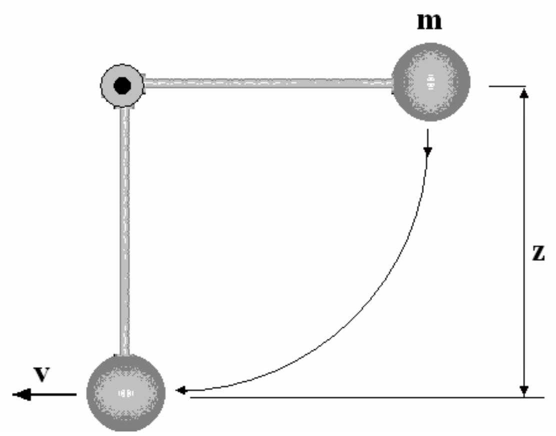


A body, which is z metres above a point, has potential energy of mgz. As it falls, the potential energy is converted in kinetic energy $mv^2/2$. If the energy conversion is perfect, then we may equate the two

$mgz = mv^2/2$ hence $v = \sqrt{(2gz)}$

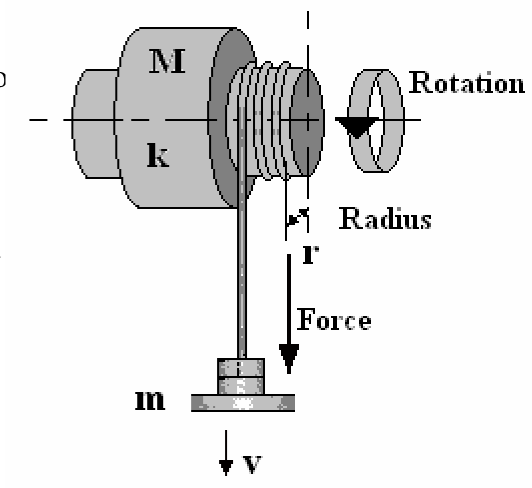
This formula may also be applied to jet of water issuing from a hole in a tank. The water is forced out by the height of water in the tank. A mass moving from the surface of the tank to the hole will lose potential energy and gain kinetic energy so the velocity is $v = \sqrt{(2gz)}$

The formula may also be applied to swinging hammers or pendulums. The hammer starts at height z and swings down so that at the bottom of the swing the kinetic energy is equal to the potential energy lost and again $\sqrt{(2gz)}$.



WORKED EXAMPLE No. 3

Consider a cylindrical body attached by a cord to a fixed point. The cord is wrapped around an axle of radius r. If the body falls, the cylinder spins and so it achieves linear and angular velocities.



Show that the velocity of the falling mass is given by

$$v^2 = \frac{2mgz}{m + I/r^2}$$

SOLUTION

P.E. lost = mgz
KE gained = $mv^2/2 + I\,\omega^2/2 = mv^2/2 + I\,\omega^2/2$
$mgz = mv^2/2 + I\,\omega^2/2$
$\omega = v/r$

$mgz = mv^2/2 + I\,v^2/2r = v^2(m/2 + I/2r^2)$ $\qquad v^2 = \dfrac{2mgz}{m + I/r^2}$

A wheel with a moment of inertia of 1.2 kg m^2 is rotated by a falling mass of 80 g attached to a string wrapped around the axle which is 40 mm diameter. Calculate the linear and angular velocity when if falls 4 m. The inertia of the axle may be ignored. Check the answer by comparing the energy before and after.

SOLUTION

$$v^2 = \frac{2mgz}{m + I/r^2} = \frac{2 \times 0.08 \times 9.81 \times 4}{0.08 + 1.2/0.02^2} = 0.0021 \qquad v = 0.045 \text{ m/s}$$

$\omega = v/r = 0.045/0.02 = 2.259$ rad/s (note the velocity of the mass and string is the same as the velocity at radius r).
P.E. Lost = mgz = 0.08 x 9.81 x 4 = 3.139 J
KE gained = $mv^2/2 + I\omega^2/2 = 0.08 \times 0.045^2/2 + 1.2 \times 2.259^2/2 = 3.139$ J

WORKED EXAMPLE No. 4

A ball of mass 20 g is dropped a height of 750 mm onto a platform resting on a spring of stiffness 50 N/m. Calculate the distance the platform will move down before the ball stops moving.

SOLUTION

Change in P. E. = mg (z + x)
Change in Strain Energy is $Kx^2/2$
$Kx^2/2 = mg\ (z + x) = mg\ z + mg\ x$
$Kx^2 = 2mg\ z + 2mg\ x$
$Kx^2 - 2mg\ x - 2mg\ z = 0$
Use the quadratic equation
a = K = 5 b = -2mg = -0.3924 c = -2mgz = 0.2943

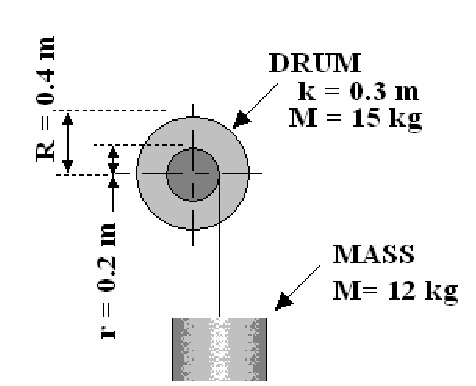
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0.3924 \pm \sqrt{-0.3924^2 - 4 \times 50 \times 0.2943}}{2 \times 50} = 0.081\text{m or} - 0.073\text{m}$$

Answer x = 81 mm

SELF ASSESSMENT EXERCISE No. 1

1. An object of mass 20 kg is dropped onto a surface from a height of 50 m. Calculate the energy and velocity just before it hits the surface.
(9810 J and 31.3 m/s)
2. A swinging hammer must have 50 Joules of energy and a velocity of 2 m/s at the bottom of the swing. Calculate the mass and height of the hammer before it is released.
(25 kg and 0.204 m)
3. A swinging hammer has a mass of 2 kg and is raised 0.2 m. Calculate the energy and velocity at the bottom of the swing.
(1.98 m/s)
4. A platform rests on springs with a stiffness of 200 kN/m. A mass of 4 kg is dropped on it from a height of 20 mm. Assuming that the deflection is small, what is the maximum distance the platform moves? (3 mm)

5. A drum is made to revolve about its centre by a falling mass as shown. Calculate the velocity of the falling mass and rotating drum when the mass descends 10 m with no frictional losses.
(13 m/s and 65.3 rad/s)



4. MECHANICAL POWER

Power is the rate of using energy or doing work. Energy has units of Joules so power is Joules per second or Watts. P = Energy used/time taken

Mechanical power is the power developed by a force as opposed to say the rate of using heat which is also a form of power.
Power = Work Done per Second = F x/t
Since x/t is the velocity v of the force then we have a definition for mechanical power. **P = F v**

5. FRICTION

Friction always causes energy to be wasted in the form of heat. It is ever present in mechanical mechanisms. When solving problems involving energy conversion, the conversion is less than 100%, the difference being the energy wasted.

EFFICIENCY

Efficiency (symbol η eta) is the ratio of the energy conversion process so

$$\eta\% = \frac{\text{Energy Out}}{\text{Energy In}} \times 100$$ and the energy wasted is Energy Out – Energy In

In devices like friction brakes, all the energy is converted into heat by friction so the efficiency does not mean anything.

WORKED EXAMPLE No. 4

A pulley is used to raise a mass of 85 kg a distance of 12 m. The efficiency is 60%. Calculate the work done.

SOLUTION

The change in P. E. = mg z = 85 x 9.81 x 12 = 10 kJ
The Work Done is the energy in so W = 10/60% = 10/0.6 = 16.67 kJ

SELF ASSESSMENT EXERCISE No. 2

1. 5 000 J of energy is used up in 20 seconds. What is the power? (250 W)
2. A vehicle is propelled 25 000 m by a force of 2 000 N in 12 seconds. Calculate the work done and the power used. (4.17 MW)
3. A block of mass 500 kg is raised at a constant rate by a hoist at a rate of 0.15 m/s. Calculate the force in the rope and hence the power used. (4905 N and 735.8 W)
4. A load is raised by a pulley. The force in the rope is 40 N and it moves 3 m in 11 seconds. The process is 70% efficient. Calculate the mechanical power. (15.6 W)
5. A rocket flies at 120 m/s under a propulsion force of 3 000 N. Calculate the power used. (360 kW)
6. A lifting jack must raise a force of 4 kN a distance of 0.3 m. Due to friction the efficiency is only 35%. Calculate the energy used to raise the load. (3429 Joules)
7. An electric hoist raises a mass of 60 kg at a rate of 0.2 m/s. The process is 30% efficient. Calculate the power input to the hoist. (392 W)