EDEXCEL NATIONAL CERTIFICATE/DIPLOMA
SCIENCE FOR TECHNICIANS

OUTCOME 1 - STATIC AND DYNAMIC FORCES

TUTORIAL 3 – STRESS AND STRAIN

1 *Static and dynamic forces*

Forces: definitions of: matter, mass, weight, force, density and relative density; pressure, scalar and vector quantities, vector representation of forces, balanced and unbalanced forces, moments of couples, conditions for equilibrium, parallelogram and triangle of forces, coplanar forces and centre of gravity.

Stress and strain*:* direct stress and strain, shear stress and strain, Hooke's law, modulus of elasticity, load extension graphs

Motion*:* force, velocity and acceleration; action and reaction, Newton's laws of motion, linear and angular motion, momentum

You should judge your progress by completing the self assessment exercises. These may be sent for marking at a cost (see home page).

On completion of this tutorial you should be able to do the following.

- Define Static and Dynamic Forces
- Define direct stress and strain.
- Define shear stress and strain.
- Define the modulus of elasticity and rigidity.
- Solve basic problems involving stress, strain and modulus.

1. **BASIC INFORMATION ABOUT FORCES**

When a force is applied to an object it may:-

- make it move.
- cause it to deform in some way.

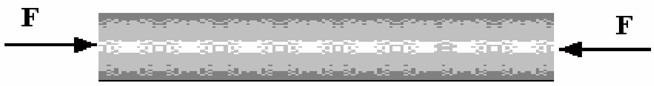
If the force causes movement, it is a DYNAMIC FORCE.

If the force does not cause movement it is STATIC FORCE.

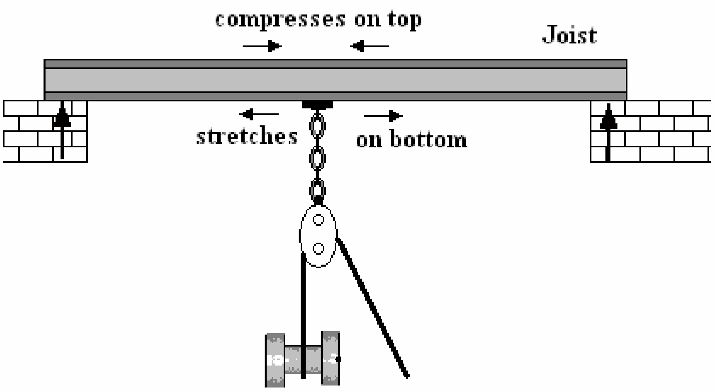
STATIC FORCES will deform a body in one or more of the following manners.

a. It may stretch the body, in which case it is called a ***TENSILE FORCE***. A rod or rope used in a frame to take a tensile load is called a ***TIE***.

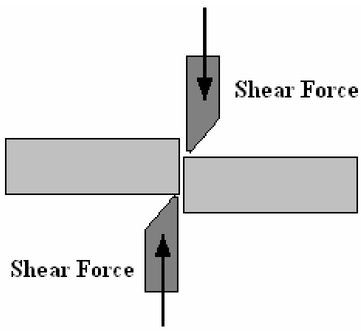
b. It may squeeze the body in which case it is called a ***COMPRESSIVE FORCE***. A part of a structure in compression may be a ***COLUMN*** and in a frame is a ***STRU***T.



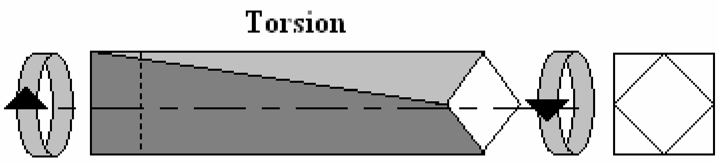
c. It may bend the body in which case both tensile and compressive forces may occur. A structure used to support a bending load is called a ***BEAM*** or ***JOIST***.



d. It may try to shear the body in which case the force is called a ***SHEAR FORCE.*** A scissors or guillotine produces shear forces.

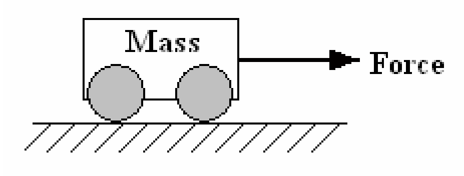


e. It may twist the body in which case ***SHEAR FORCES*** occur. A structure that transmits rotation is called a ***SHAFT*** and it experiences ***TORSION***.

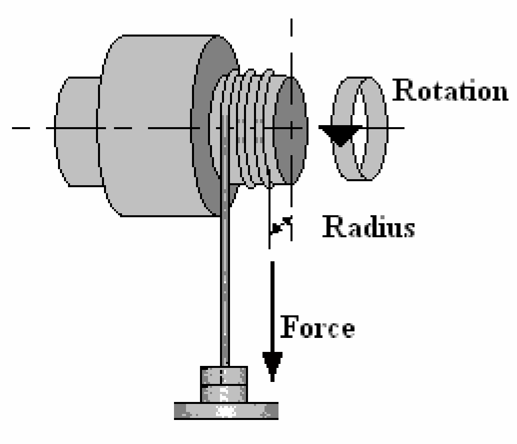


DYNAMIC FORCES may cause the body to move in the following manner.

a. It may make the body move in a straight line in which case we have ***LINEAR MOTION.***



b. It may make the body rotate in which case we have ***ROTARY MOTION*** or ***ANGULAR MOTION***.



TORQUE

I
n order to make the body rotate the force must act a radius from the centre and exert a TORQUE on the body.

T = F x Radius

WORK AND POWER

When a force moves, it does **WORK.**
The definition of work is as follows.
Work = Force x distance moved or W = F x

The units of work are Newton x metre or N m. Since work is equivalent to energy we may say that energy is stored work. The unit of energy is the Joule so it follows that one Nm is a Joule.

The energy used or the work done in one second is called the **POWER**. Power = Work/time taken.
Power units are Joules/second and this is called the Watt.

SELF ASSESSMENT EXERCISE No. 1

1. A rocket engine produces a thrust of 5000 N and the rocket moves 1200 m in 3 seconds. Calculate the work done and the power of the engine.

2. A crane lifts a weight of 3000 N a height of 30 m at a steady speed in 20 seconds. Calculate the work done and the power of the winch.

3. Explain why a rope or chain could never be used to make a strut.

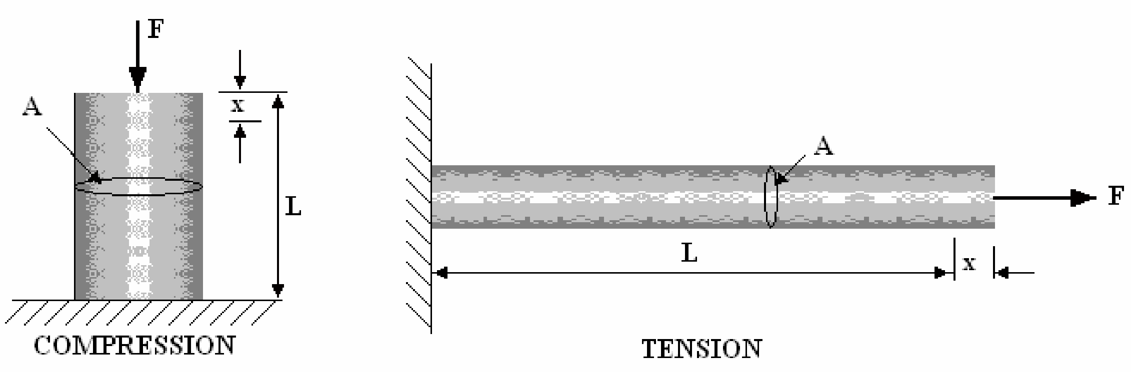
4. A lintel is placed over a door or window to support the bricks above. Is a lintel a beam or a column?

5. Is the rope used in crane a strut or a tie.

6. Does the shaft of a motor experience bending or torsion?

2. DIRECT STRESS σ

When a force is applied to an elastic body, the body deforms. The way in which the body deforms depends upon the type of force applied to it.



A compression force makes the body shorter. A tensile force makes the body longer.

Tensile and compressive forces are called **DIRECT FORCES.**
Stress is the force per unit area upon which it acts.

Stress = σ = Force/Area N/m^2 or Pascals. Note the cross section may be any shape.

The symbol σ is called SIGMA

NOTE ON UNITS The fundamental unit of stress is 1 N/m^2 and this is called a Pascal. This is a small quantity in most fields of engineering so we use the multiples kPa, MPa and GPa.

Areas may be calculated in mm^2 and units of stress in N/mm^2 are quite acceptable. Since 1 N/mm^2 converts to 1 000 000 N/m^2 then it follows that the N/mm^2 is the same as a MPa

3. DIRECT STRAIN ε

In each case, a force F produces a deformation x. In engineering we usually change this force into stress and the deformation into strain and we define these as follows.

Strain is the deformation per unit of the original length

Strain = ε = x/L

The symbol ε is called EPSILON

Strain has no units since it is a ratio of length to length. Most engineering materials do not stretch very much before they become damaged so strain values are very small figures. It is quite normal to change small numbers in to the exponent for of 10^{-6}. Engineers use the abbreviation με (micro strain) to denote this multiple.

For example a strain of 0.000068 could be written as 68 x 10^{-6} but engineers would write 68 με.

Note that when conducting a British Standard tensile test the symbols for original area are S_o and for Length is L_o.

WORKED EXAMPLE No. 1

A metal wire is 2.5 mm diameter and 2 m long. A force of 12 N is applied to it and it stretches 0.3 mm. Assume the material is elastic. Determine the following.
i. The stress in the wire σ.
ii. The strain in the wire ε.

SOLUTION

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 2.5^2}{4} = 4.909\ \text{mm}^2 \qquad \sigma = \frac{F}{A} = \frac{12}{4.909} = 2.44\ \text{N/mm}^2$$

Answer (i) is hence 2.44 MPa

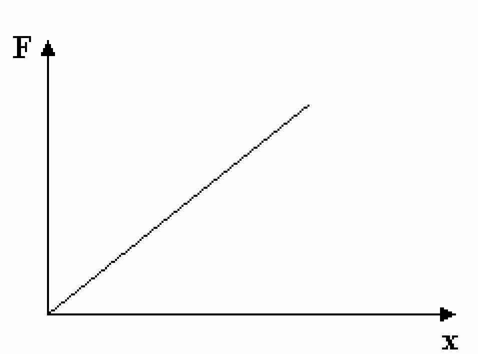
$$\varepsilon = \frac{x}{L} = \frac{0.3\ \text{mm}}{2000} = 0.00015 \text{ or } 150\ \mu\varepsilon$$

SELF ASSESSMENT EXERCISE No.2

1. A steel bar is 10 mm diameter and 2 m long. It is stretched with a force of 20 kN and extends by 0.2 mm. Calculate the stress and strain.
   (Answers 254.6 MPa and 100 με)

2. A rod is 0.5 m long and 5 mm diameter. It is stretched 0.06 mm by a force of 3 kN. Calculate the stress and strain.
   (Answers 152.8 MPa and 120με)

4. MODULUS OF ELASTICITY E



Elastic materials always spring back into shape when released. They also obey HOOKE'S LAW. This is the law of a spring which states that deformation is directly proportional to the force. F/x = stiffness = k N/m

The stiffness is different for different materials and different sizes of the material. We may eliminate the size by using stress and strain instead of force and deformation as follows. If F and x refer to direct stress and strain then

$F = \sigma A \quad x = \varepsilon L$ hence $\frac{F}{x} = \frac{\sigma A}{\varepsilon L}$ and $\frac{FL}{Ax} = \frac{\sigma}{\varepsilon}$

The stiffness is now in terms of stress and strain only and this constant is called the **MODULUS of ELASTICITY and it has a symbol E.**

$$E = \frac{FL}{Ax} = \frac{\sigma}{\varepsilon}$$

A graph of stress against strain will be a straight line with a gradient of E. The units of E are the same as the units of stress.

5. ULTIMATE TENSILE STRESS

If a material is stretched until it breaks, the tensile stress has reached the absolute limit and this stress level is called the ultimate tensile stress. Values for different materials may be found in various sources such as the web site Matweb.

WORKED EXAMPLE No. 2

A steel tensile test specimen has a cross sectional area of 100 mm^2 and a gauge length of 50 mm, the gradient of the elastic section is 410 x 10^3 N/mm. Determine the modulus of elasticity.

SOLUTION

The gradient gives the ratio F/A = and this may be used to find E.

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{x} x \frac{L}{A} = 410 \times 10^3 \times \frac{50}{100} = 205\,000 \text{ N/mm}^2 \text{ or } 205\,000 \text{ MPa or } 205 \text{ GPa}$$

WORKED EXAMPLE No. 3

A Steel column is 3 m long and 0.4 m diameter. It carries a load of 50 MN. Given that the modulus of elasticity is 200 GPa, calculate the compressive stress and strain and determine how much the column is compressed.

SOLUTION

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.4^2}{4} = 0.126 \text{ m}^2 \qquad \sigma = \frac{F}{A} = \frac{50 \times 10^6}{0.126} = 397.9 \times 10^6 \text{ Pa}$$

$$E = \frac{\sigma}{\varepsilon} \quad \text{so} \quad \varepsilon = \frac{\sigma}{E} = \frac{397.9 \times 10^6}{200 \times 10^9} = 0.001989$$

$$\varepsilon = \frac{x}{L} \quad \text{so} \quad x = \varepsilon L = 0.001989 \times 3000 \text{ mm} = 5.97 \text{ mm}$$

SELF ASSESSMENT EXERCISE No. 3

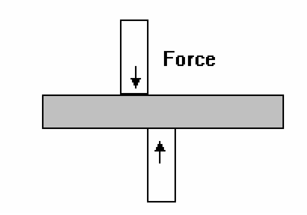
1. A bar is 500 mm long and is stretched to 505 mm with a force of 50 kN. The bar is 10 mm diameter. Calculate the stress and strain.
   The material has remained within the elastic limit. Determine the modulus of elasticity.
   (Answers 636.6 MPa, 0.01 and 63.66 GPa.)

2. A steel bar is stressed to 280 MPa. The modulus of elasticity is 205 GPa. The bar is 80 mm diameter and 240 mm long.
   Determine the following.
   i. The strain. (0.00136)
   ii. The force. (1.407 MN)

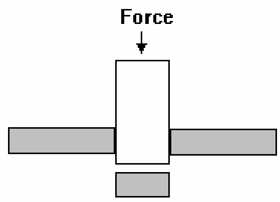
3. A circular metal column is to support a load of 500 Tonne and it must not compress more than 0.1 mm. The modulus of elasticity is 210 GPa. the column is 2 m long.
   Calculate the cross sectional area and the diameter. (0.467 m^2 and 0.771 m)
   Note 1 Tonne is 1000 kg.

6. SHEAR STRESS τ

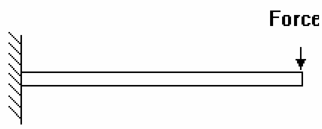
Shear force is a force applied sideways on to the material (transversely loaded). This occurs typically :



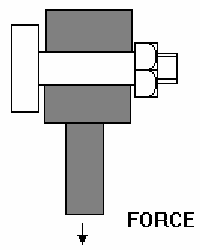
Shearing



Punching



Transverse Force

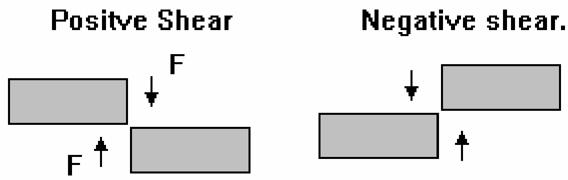


Shear Pin

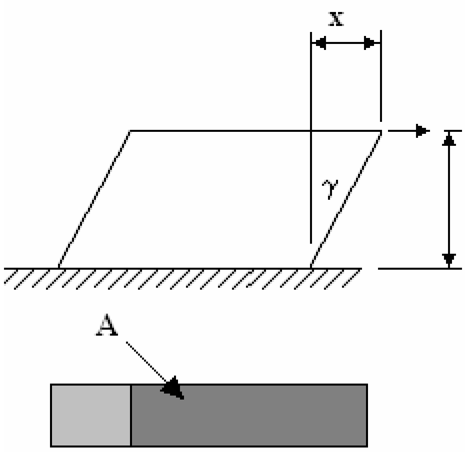
Shear stress is the force per unit area carrying the load. This means the cross sectional area of the material being cut, the beam and pin respectively.

Shear stress $\tau = F/A$ The symbol τ is called Tau

The sign convention for shear force and stress is based on how it shears the materials and this is shown below.



In order to understand the basic theory of shearing, consider a block of material being deformed sideways as shown.



7. SHEAR STRAIN γ

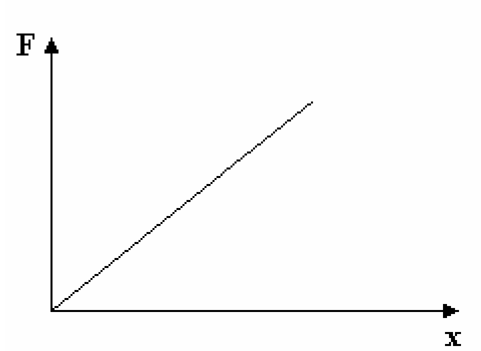
The force causes the material to deform as shown. The shear strain is defined as the ratio of the distance deformed to the height x/L.

The end face rotates through an angle γ. Since this is a very small angle, it is accurate to say the distance x is the length of an arc of radius L and angle γ so that

$$\gamma = x/L$$

It follows that γ is the shear strain. The symbol γ is called Gamma.

8. MODULUS OF RIGIDITY G



If we were to conduct an experiment and measure x for various values of F, we would find that if the material is elastic, it behave like a spring and so long as we do not damage the material by using too big a force, the graph of F and x is a straight line as shown.
The gradient of the graph is constant so F/x = constant and this is the spring stiffness of the block in N/m.

If we divide F by the area A and x by the height L, the relationship is still a constant and we get

$$\frac{F}{A} \div \frac{x}{L} = \frac{FL}{Ax} = \text{constant}$$

But F/A = τ and x/L = γ so $\frac{F}{A} \div \frac{x}{L} = \frac{FL}{Ax} = \frac{\tau}{\gamma} = \text{constant}$

This constant will have a special value for each elastic material and is called the Modulus of Rigidity with symbol G.

$$\frac{\tau}{\gamma} = G$$

9. ULTIMATE SHEAR STRESS

If a material is sheared beyond a certain limit it becomes permanently distorted and does not spring all the way back to its original shape. The elastic limit has been exceeded. If the material is stressed to the limit so that it parts into two(e.g. a guillotine or punch), the ultimate limit has been reached. The ultimate shear stress is τ_u and this value is used to calculate the force needed by shears and punches.

WORKED EXAMPLE No. 4

Calculate the force needed to guillotine a sheet of metal 5 mm thick and 0.8 m wide given that the ultimate shear stress is 50 MPa.

SOLUTION

The area to be cut is a rectangle 800 mm x 5 mm

$A = 800 \text{ x } 5 = 4000 \text{ mm}^2$ The ultimate shear stress is 50 N/mm^2

$$\tau = \frac{F}{A} \quad \text{so} \quad F = \tau \text{ x } A = 50 \text{ x } 4000 = 200\,000 \text{ N or } 200 \text{ kN}$$

WORKED EXAMPLE No. 5

Calculate the force needed to punch a hole 30 mm diameter in a sheet of metal 3 mm thick given that the ultimate shear stress is 60 MPa.

SOLUTION

The area to be cut is the circumference x thickness = $\pi d \text{ x } t$

$A = \pi \text{ x } 30 \text{ x } 3 = 282.7 \text{ mm}^2$ The ultimate shear stress is 60 N/mm^2

$\tau = \frac{F}{A}$ so $F = \tau \text{ x } A = 60 \text{ x } 282.7 = 16965$ N or 16.965 kN

WORKED EXAMPLE No. 6

Calculate the force needed to shear a pin 8 mm diameter given that the ultimate shear stress is 60 MPa.

SOLUTION

The area to be sheared is the circular area $A = \frac{\pi d}{4}$

$A = \frac{\pi \text{ x } 8^2}{4} = 50.26 \text{mm}^2$ The ultimate shear stress is 60 N/mm^2

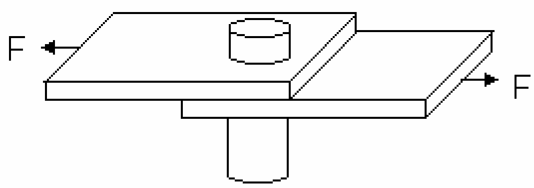
$\tau = \frac{F}{A}$ so $F = \tau \text{ x } A = 60 \text{ x } 50.26 = 3016$ N or 3.016 kN

SELF ASSESSMENT EXERCISE No. 4

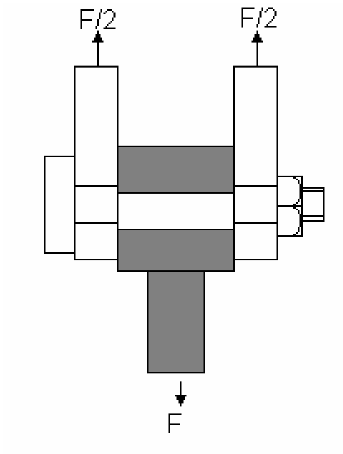
1. A guillotine must shear a sheet of metal 0.6 m wide and 3 mm thick. The ultimate shear stress is 45 MPa. Calculate the force required. (Answer 81 kN)

2. A punch must cut a hole 30 mm diameter in a sheet of steel 2 mm thick. The ultimate shear stress is 55 MPa. Calculate the force required. (Answer10.37 kN)

3. Two strips of metal are pinned together as shown with a rod 10 mm diameter. The ultimate shear stress for the rod is 60 MPa. Determine the maximum force required to break the pin. (Answer 4.71 kN)



10. DOUBLE SHEAR



Consider a pin joint with a support on both ends as shown. This is called a CLEVIS and CLEVIS PIN. If the pin shears it will do so as shown.

By balance of forces, the force in the two supports is F/2 each.

The area sheared is twice the cross section of the pin so it takes twice as much force to break the pin as for a case of single shear. Double shear arrangements doubles the maximum force allowed in the pin.

WORKED EXAMPLE No. 7

A pin is used to attach a clevis to a rope. The force in the rope will be a maximum of 60 kN. The maximum shear stress allowed in the pin is 40 MPa. Calculate the diameter of a suitable pin.

SOLUTION

The pin is in double shear so the shear sress is $\tau = \frac{F}{2A}$

$$A = \frac{F}{2\tau} = \frac{60000}{2 \text{ x } 40 \text{x} 10^6} = 750 \text{ x } 10^{-6} \text{ m}^2$$

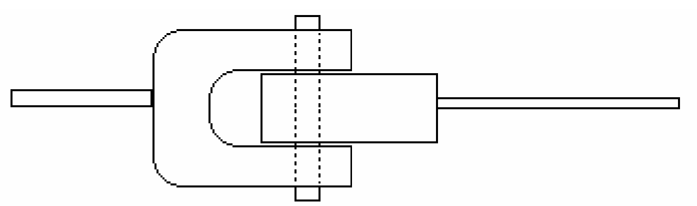
$$A = 750 \text{ mm}^2 = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{4 \text{ x } 750}{\pi}} = 30.9 \text{ mm}$$

SELF ASSESSMENT EXERCISE No. 5

1. A clevis pin joint as shown above uses a pin 8 mm diameter. The shear stress in the pin must not exceed 40 MPa. Determine the maximum force that can be exerted. (Answer 4.02 kN)

2.



A rope coupling device shown uses a pin 5 mm diameter to link the two parts. If the shear stress in the pin must not exceed 50 MPa, determine the maximum force allowed in the ropes. (Answer 1.96 kN)