EDEXCEL NATIONAL CERTIFICATE/DIPLOMA
SCIENCE FOR TECHNICIANS

OUTCOME 1 - STATIC AND DYNAMIC FORCES

TUTORIAL 2 – FORCE VECTORS

1 Static and dynamic forces

Forces: definitions of: matter, mass, weight, force, density and relative density; pressure, scalar and vector quantities, vector representation of forces, balanced and unbalanced forces, moments of couples, conditions for equilibrium, parallelogram and triangle of forces, coplanar forces and centre of gravity.

Stress and strain*:* direct stress and strain, shear stress and strain, Hooke's law, modulus of elasticity, load extension graphs

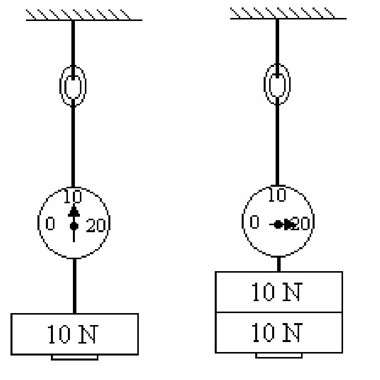
Motion*:* force, velocity and acceleration; action and reaction, Newton's laws of motion, linear and angular motion, momentum

You should judge your progress by completing the self assessment exercises. These may be sent for marking at a cost (see home page).

On completion of this tutorial you should be able to do the following.

- Define vector and scalar quantities.
- Represent forces as vectors.
- Draw the triangle and polygon of force to solve an unknown force,
- Define equilibrium and solve equilibrant forces.
- Define the centre of gravity of a body.

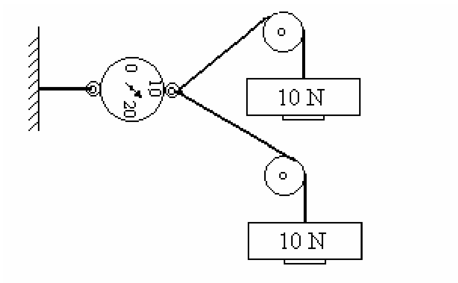
1. **INTRODUCTION**



When we use ordinary numbers we can add them, subtract them, and multiply them and so on.

Consider a weighing scale as shown. If we put a 10 N on the hanger the instrument shows 10 N. If we put another 10 N on the instrument indicates 20 N being the sum 10 + 10 as we would expect.

This is not always the case.

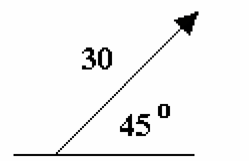


Now consider the following system. When two 10 N weights are hung on the hangers, the instrument reads 15 N not 20 N. This is because the two weights are no longer pulling in the same direction but in two different directions. Clearly when the direction is important, we need a different method of adding them together. This is one reason why we use vectors.

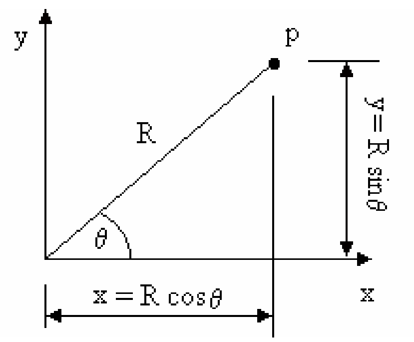
2. VECTOR and SCALAR DEFINITIONS

A vector may represent anything that has magnitude (size) and direction. In this module we are only concerned with FORCE.

If the quantity has magnitude and no direction, it is called a SCALAR. Examples are temperature and density.



In order to represent a force as a vector, we draw an arrow with the length proportional to the force and the direction the same as the true direction of the force. The diagram shows a vector representing 30 N at 45^o.



Let’s revise the ways of representing a point on a graph. You should be familiar with ***CARTESIAN*** and ***POLAR*** co-ordinates.

Point p is at Cartesian coordinates x and y or polar coordinates R and θ. Note θ is measured anticlockwise from the positive x axis. The two systems are clearly linked as we can convert from one to the other using trigonometry and Pythagoras’ theorem.

$y = R \sin\theta \qquad x = R \cos\theta \qquad R = (x^2 + y^2)^{½}$

3. VECTOR ADDITION and SUBTRACTION

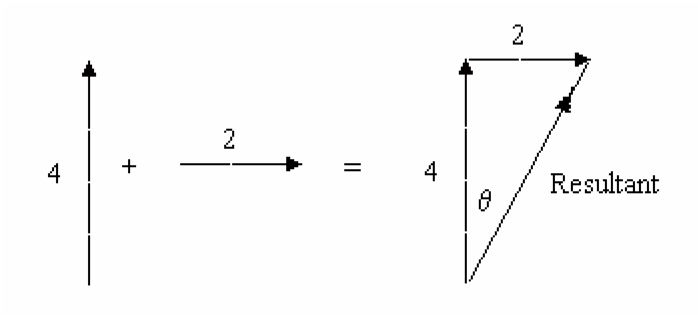
When two forces act at a point, the total force and its true direction are found by adding them as vectors. We do not add the values of the forces.

To do this we draw the first vector (it doesn't matter which one) and then draw the second starting on the tip of the first. The new vector which starts at the tail of the first and ends at the head of the second is the resultant force vector.

In this module, we are only dealing with forces that act on the same 2 dimensional plane and these are said to be COPLANAR.

WORKED EXAMPLE No. 1

Determine the result of adding two coplanar forces
4 N at 90^o and 2 N at 0^o.



SOLUTION

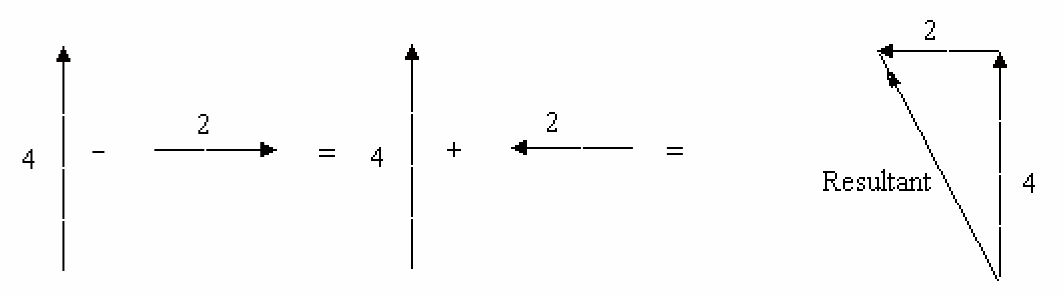
The magnitude and direction of the resultant may be found graphically by drawing it all out to scale and measuring it, or by trigonometry. In this case it is a right angle triangle so use Pythagoras.

$R^2 = 2^2 + 4^2 = 4 + 16 = 20$
$R = \sqrt{20} = 4.47$
$\tan\theta = 2/4 = 0.5$
$\theta = 26.6^o$

WORKED EXAMPLE No. 2

Subtract the coplanar forces of 2 at 0^o from a vector of 4 at 90^o.

SOLUTION



RESOLUTION METHOD

All coplanar vectors may be resolved into horizontal and vertical components using

$x = R\cos\theta \qquad y = R\sin\theta$

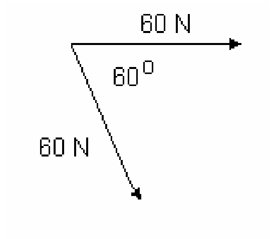
All the components may be then added to give the total vertical and horizontal components of the resultant. This is found from Pythagoras

$$R = (x^2 + y^2)^{1/2}$$

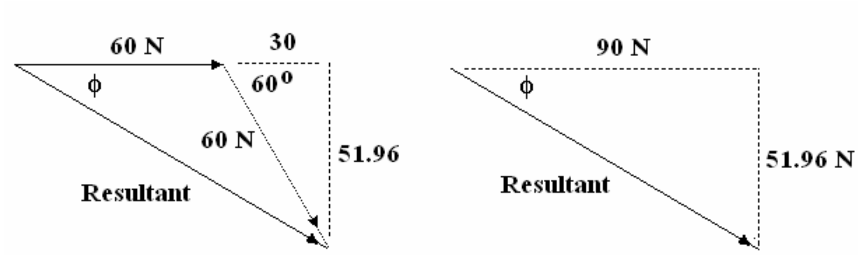
Where x and y are the totals.

WORKED EXAMPLE No. 3

Add the two coplanar forces shown.



SOLUTION



Draw the vectors as shown (not to scale here) and measure the resultant and the angle. You should get 103.9 N and 30°.

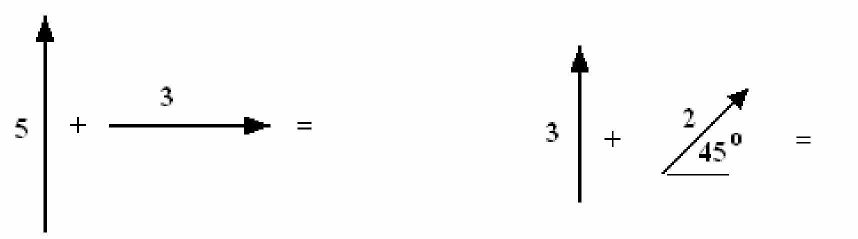
Alternatively resolve the vectors vertically and horizontally and add them to get the coordinates of the resultant. Note the angle is shown clockwise so it is -60°.

Resolving the first gives 60 N horizontal and 0 N vertical. Resolving the other we get

Vertical = 60 sin (-60°) = -51.96 N Horizontal = 60 cos (-60°)= 30 N
Total vertical = 51.96 Total horizontal = 90 N
Resultant = $\sqrt{(0^2 + 51.96^2)}$ = 103.9 N $\phi = \tan^{-1} 51.96/90 = 30°$

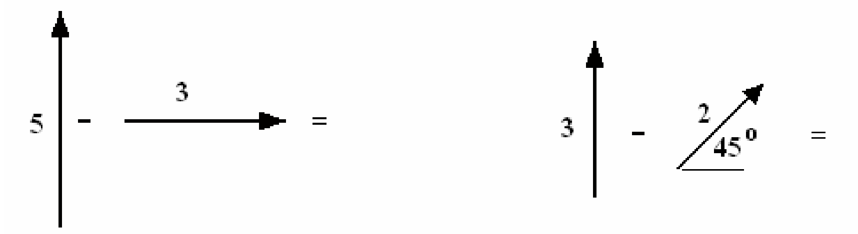
SELF ASSESSMENT EXERCISE No. 1

1. Determine the result of the vector additions shown.



(Answers 5.831 at 59° and 4.64 at 72.2°.)

2. Find the resultant vector for the cases shown.



(Anwers 5.83 at 121° and 4.64 at 107.8°)

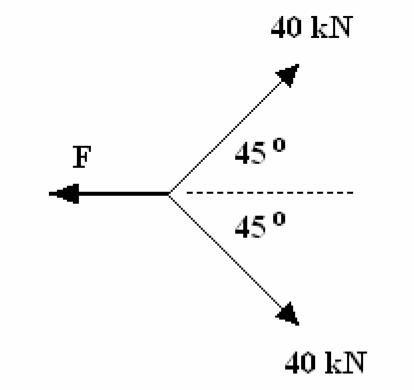
4. EQUILIBRIUM OF FORCES

When several forces act at a point and that point does not move, then the forces must be completely balanced and the point is said to be in equilibrium. In other words there is no resultant force.

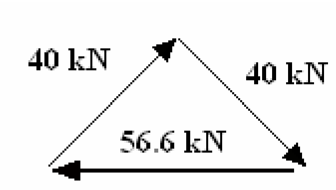
Consider three forces acting on point as shown. The point is in balance. Add the vectors up and show that there is no resultant.

WORKED EXAMPLE No. 4

Find the force F required to balance the two forces shown.



SOLUTION



Draw the two forces as shown to scale. The vector that closes the triangle is the equilibrium force and it is horizontal with a value of 56.6 kN.

Using resolution :

Horizontal component = 40 cos (45^o) + 40 cos (-45^o) = 56.6 kN
Vertical component = 40 sin 45^o + 40 sin (-45^o) = 0 kN

5. POLYGON OF FORCES

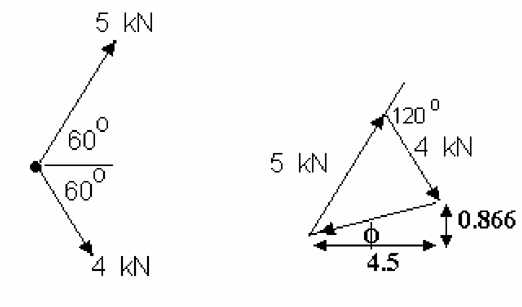
If a body is in equilibrium and one of the forces is unknown, we may find it because we know that the force vectors must have no resultant. The unknown force will be the one which closes the vector diagram.

If there are three forces, the unknown force completes a triangle and it is called a TRIANGLE OF FORCE. If there are more forces we have a POLYGON OF FORCES.

The force which closes the vector diagram is equal and opposite to the resultant of the known forces and this is called the EQUILIBRANT FORCE.

WORKED EXAMPLE No. 5

Find the equilibrium force required to balance the two forces shown.



SOLUTION

Draw to scale as shown and the equilibrant is 4.58 kN at 11o to the horizontal as shown.

Using resolution:- Vertical component 5 sin 60° + 4 sin (-60°) = 0.866 kN

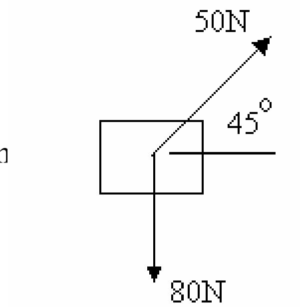
Horizontal component 5 cos 60° + 4 cos (-60°) = 4.5 kN

$F = \sqrt{(0.866^2 + 4.5^2)} = 4.58$ kN

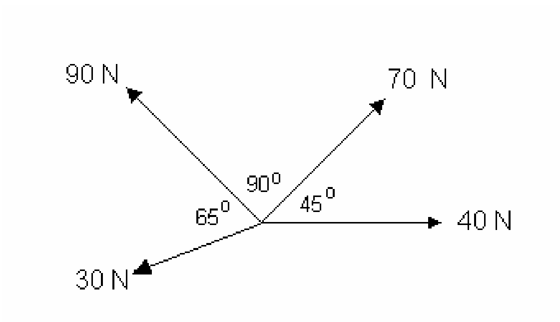
$\phi = \tan^{-1} 0.866/4.5 = 10.9°$

SELF ASSESSMENT EXERCISE No. 2

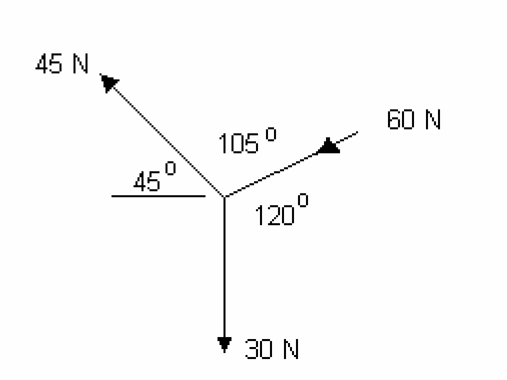
1. Draw a triangle of forces and determine the force which holds the body shown below in equilibrium



2. Find the extra force needed to balance the system shown below.



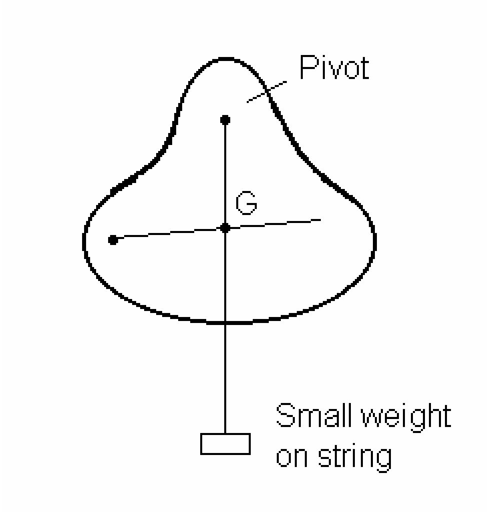
3. Determine the fourth force needed to hold the three shown below in balance.



6. CENTRES OF GRAVITY

The centre of gravity of a body is the point where it would balance if it was supported at that point. In other words all the forces due to the weight would be in equilibrium.

If we suspended the body on a pivot as shown, the centre of gravity will come to rest vertically below the pivot. If we did this about two pivots, the centre of gravity is where the lines intersect as shown and usually denoted with a G.



In a 3 dimensional body this point is somewhere inside it. We would need to suspend it from two points that are not co-planar.

In dynamics, the centre of gravity is the point at which we often assume the mass to act as though it was all concentrated at that point. This is a simplification that can lead to serious errors in some types of problems.